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FROM ANALYSIS OF LUNAR ORBITER TRACKING DATA

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ABSTRACT

Tracking data from the Lunar Orbiter spacecraft are being analyzed at Langley Research Center to define the coefficients in the expansion of the lunar gravitational field in terms of spherical harmonics. The primary basis for the analysis of the gravitational components and other parameters of interest, is a special purpose computational approach using numerical techniques and the procedures of differential correction and weighted least squares. A preliminary set of gravitational field coefficients (through fifth degree and fifth order) is presented, and the contributions and possible shortcomings of this set are discussed. Contour plots of lunar topography, as derived from the gravitational field results, are presented. With respect to the mass distribution in the moon, the initial indications are that the moment of inertia about the polar axis is greater than that which would be expected on the basis of a homogeneous density distribution.

INTRODUCTION

A basic objective of the analysis of the tracking data from the Lunar Orbiter series of spacecraft is to define the components of the gravitational field of the moon, for application to determination of various physical properties of the moon and for application to orbit prediction for lunar satellites. The specific objective is the determination of the coefficients $C_{n,m}$ and $S_{n,m}$ of a finite number of terms in the infinite series expansion of the lunar gravitational potential function in spherical harmonics

$$U = \frac{\mu}{r} \left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^n P_{n,m}(\sin \phi) (C_{n,m} \cos \lambda m + S_{n,m} \sin \lambda m) \right]$$

where μ is the product of the gravitational constant and the mass of the moon, a is the mean radius of the moon, r is the radial distance from the center of mass of the moon, $P_{n,m}$ are the associated Legendre polynomials, ϕ is latitude, (measured with respect to the lunar equator) and λ is longitude. The form of the potential function adopted here is that recommended by the International Astronomical Union.

In the research effort for determination of the components of the lunar gravitational field at the NASA Langley Research Center, two different approaches are currently in use. Both approaches are based

on the procedures of iterative differential corrections, weighted least squares, and numerical integration of the equations of motion of the lunar spacecraft. In the primary procedure, which is designated as the "direct" approach, the observational equations are formulated directly in terms of the tracking data observable, and the partial derivatives required in the differential correction process are the partial derivatives of the observables with respect to the parameters to be determined. The numerical integration of the Cowell-type equations of motion is performed using a high-accuracy, 12th order predictor-corrector technique with a 4th order starting procedure. In the other approach, which can be designated as a "long-period" approach, the mean elements of the orbit are derived at daily intervals from a fit to about two orbits of tracking data each day using the direct procedure, and the mean elements are taken as the basic data in the observational equations, in which the partial derivatives relate the mean elements to the parameters to be determined. At the present stage of the analysis, more data have been analyzed by the direct approach than by the long-period approach, and the results presented here are from the direct approach.

The results and discussions presented in this paper should be considered as interim results which may be subject to modification, perhaps considerable modification, when additional data become available. The cautionary attitude arises from consideration of inherent limitations

in the data which are available for the analyses to date, limitations primarily associated with the relatively low inclination of the lunar satellite orbital plane with respect to the lunar equator. Whereas the early earth satellites had inclinations of 35° or more, most of the data included in the present analysis are from lunar satellites with inclinations of 12° and 17.5° , which means that the results are biased by gravitational effects with origin in the equatorial region of the moon. Nevertheless, with this limitation implied throughout the discussion, it is of interest to apply the gravitational field results presented here to calculations of preliminary results for the polar moment of inertia of the moon, the topography of the moon, and related items, and to compare these results with information which was previously available.

DATA USED IN THE ANALYSIS

Three Lunar Orbiter Spacecraft have been successfully injected into orbit about the moon, the second and third at three month intervals from the first, which was launched in August 1966 (cf. Michael, Tolson and Gapcynski (1966)). Chronological events for each of the spacecraft are summarized in Table I. Tracking data from each of these spacecraft have been used for the analysis of the gravitational field parameters presented in this paper. The observable is counted two-way doppler

frequency, averaged over time intervals of one minute or less. The observable is a quantity which is proportional to the relative line of sight velocity between the tracking station and the spacecraft. Other data types are available but these have not been used in this analysis. The tracking data are obtained through the facilities of the NASA Deep Space Network, operated for NASA by the Jet Propulsion Laboratory, using tracking stations at Goldstone, Woomera, and Madrid.

The specific data arcs used for the analysis are summarized in Table II, along with other pertinent information. The data are generally well distributed throughout the orbits, except during occultations by the moon which amount to about 45 minutes during the orbital period of about 3 1/2 hours. Data obtained during the photographic phases of the missions are not analyzed because of frequent spacecraft maneuvers which possibly introduce small translational accelerations on the spacecraft due to uncoupled attitude control system jets. (These small perturbations are not accounted for in the data arcs used, but they may be considered in future work if necessary. Some limited post-flight analyses indicate that the effects are probably negligible.) The 24-day data arc of orbit configuration I-2 and the 21-day data arc of II-1 represent large fractions of complete revolutions of the moon. The orbital elements shown in Table II do not represent a wide variation in these properties, which is one of the limitations on the data available

for analysis at this time. For the five data arcs, pericenter is never greater than 11.5 degrees from the lunar equator (the latitude of pericenter is -11.5 degrees at the end of arc I-2, and is less than ± 5 degrees for the other arcs). The data near pericenter can be expected to exhibit the most pronounced influence of the gravitational field components for the orbital parameters of these data arcs, and thus the results obtained with these data are strongly influenced by gravitational effects with origin in the equatorial regions.

PRELIMINARY RESULTS FOR THE GRAVITATIONAL FIELD OF THE MOON

Values for the lunar gravitational field coefficients through fifth degree and order, as obtained from analysis of tracking data over the data arcs discussed above, are presented in Table III. All the coefficients listed in the table were included in the solution except for $C_{5,0}$ which is very highly correlated with $C_{3,0}$, and which was set to zero. For these results the gravitational constant of the moon, μ , is $4902.58 \text{ km}^3/\text{sec}^2$ and the mean radius is assumed to be 1738.09 km.

The standard deviations on the coefficients are more an indication of how the solution fits the data than of changes which may be expected in future analyses. These standard deviations do not include any effects of biases produced by higher degree and order coefficients which were not included in this analysis. Because the available data are limited to the

equatorial region of moon, correlations between estimated and non-estimated coefficients may be as large as those discussed below and hence the additional biases may be substantial. The second degree coefficients, which are of interest with respect to the moments of inertia of the moon, are relatively poorly determined and are fairly highly correlated among themselves and with a number of the higher degree coefficients. For instance, from correlation matrices obtained with the solution, $C_{2,0}$ is highly correlated with $C_{4,0}$ (.997), with $C_{2,1}$ (-.79) and with $C_{4,1}$ (-.83); $C_{2,1}$ with $C_{4,1}$ (.96); $C_{2,2}$ with $C_{4,2}$ (.85); $S_{2,1}$ with $S_{4,1}$ (.96); $S_{2,2}$ with $S_{4,2}$ (.88); and various lower correlations exist among these and other coefficients. These high correlations were anticipated from pre-flight analyses (cf. Michael and Tolson (1965), and Tolson and Compton (1966)), and indicate that it is difficult at this stage of the analysis to obtain separation of coefficients which produce similar orbital perturbations on the spacecraft. The separation is best obtained through analysis of tracking data from spacecraft with a variety of orbital parameters, and particularly with higher inclination orbits.

On comparing the gravitational coefficients for the moon and the earth, the oblateness term, $C_{2,0}$, is greater for the earth (-1.08×10^{-3}) than for the moon (-2.22×10^{-4}), as expected. For the higher degree

and order coefficients, the quantities

$$\sigma_{n,m} = \left[C_{n,m}^2 + S_{n,m}^2 \right]^{1/2}$$

for the moon are compared with those for the earth, using the earth coefficients derived by Anderle (1965). The values for $\sigma_{n,m}$ for the moon are larger than those for the earth, by factors of about 10 to 100, with the larger factor applying to the higher degree terms. For $m = 2, 3$, and 4 , the quantities decrease with increasing n in roughly the same manner for both the earth and the moon. Although this comparison indicates that the gravitational field of the moon is somewhat "rougher" than that of the earth, the relatively large values for the lunar coefficients may be biased by the correlations and the inclusion of effects due to neglected higher harmonics, as mentioned above.

APPLICATIONS TO MASS DISTRIBUTION IN THE MOON

The second degree coefficients in the gravitational potential expansion can be related to the moments and products of inertia of the moon, to show what the present results indicate with respect to the mass distribution in the moon. Axes fixed in the moon are defined as a cartesian set X , Y , and Z , with the X and Y axes in the equatorial plane of the moon, the X axis directed toward the mean

direction to the earth, and the Z axis directed northward along the axis of rotation. With the definitions for the moments and products of inertia about these body axes,

$$A = I_{xx}$$

$$B = I_{yy}$$

$$C = I_{zz}$$

$$D = I_{yz}$$

$$E = I_{xz}$$

$$F = I_{xy}$$

the relations between the gravitational coefficients and these quantities are

$$C_{2,1} = \frac{E}{Ma^2}$$

$$S_{2,1} = \frac{D}{Ma^2}$$

$$S_{2,2} = \frac{1}{2} \frac{F}{Ma^2}$$

$$C_{2,0} = \frac{1}{Ma^2} \left[\frac{A+B}{2} - C \right]$$

$$C_{2,2} = \frac{1}{4Ma^2} (B-A)$$

where M is the mass of the moon and a is its mean radius.

For later use, the quantities L and K are defined and are related to the coefficients as follows:

$$L = \frac{3}{2} \frac{(C-A)}{Ma^2} = 3 C_{2,2} - \frac{3}{2} C_{2,0}$$

$$K = \frac{3}{2} \frac{(B-A)}{Ma^2} = 6 C_{2,2}$$

There are only 5 second degree coefficients to be related to the 6 quantities defining the moments and products of inertia, so that while the products of inertia are defined by particular coefficients, the moments of inertia appear as differences and another relation is required to define the individual values, as discussed below.

Results from determination of the differences in moments of inertia of the moon by classical procedures through analyses of optical and physical librations, and of the motions of the node and perigee of the moon's orbit, serve as a basis for comparison with some of the results obtained from the gravitational field parameters. A summary of some of the previous results is presented in Table IV. The definitions of the various parameters, shown on the bottom of the table, are those given by Jeffries (1959). Values obtained by Jeffries (1961) from optical and physical librations, and by Cook (1959) and Jeffries (1961) from the motion of node and perigee are taken as one set for comparison, and the more recent results of Koziel (1967) and Eckert (1965), from the respective determinations, are taken as another set. The earlier results indicate values of f , the mechanical ellipticity, above the critical value of 0.662, while the later results are in essential agreement, with values below the critical value. The parameter g , related to the polar moment of inertia, can be obtained in two ways,

from L and β , and from K and γ , although β is generally considered to be the more precisely determined parameter. Of the four values for g shown in Table IV, three imply a polar moment of inertia approaching that of a thin spherical shell (the "hollow moon" paradox). No value for g is close to the value of 0.5956 suggested by Jeffries (1959) on the basis of reasonable hypotheses on the internal composition of the moon, which value is close to what would be obtained for a homogeneous density distribution, 0.6. For comparison with the results shown in Table III, assuming $g = 0.6$, the values $C_{2,0} = -2.10 \times 10^{-4}$ and -2.05×10^{-4} , and $C_{2,2} = 0.205 \times 10^{-4}$ and 0.231×10^{-4} are derived from β and γ as given in Table IV.

For the values of $C_{2,0}$ and $C_{2,2}$ given in Table III, there are obtained

$$L = 4.289 \times 10^{-4}$$

$$K = 1.920 \times 10^{-4}$$

$$\text{and } f = 1 - \frac{K}{L} = 0.552$$

In order to calculate a relation for the polar moment of inertia, the value for β ,

$$\beta = \frac{C - A}{C} = 6.28 \times 10^{-4}$$

is adopted, with which

$$g = \frac{3}{2} \frac{C}{Ma^2} = \frac{L}{\beta} = 0.683$$

This value for g is considerably less than three of the values shown in Table IV, but is still rather large compared with the result for homogeneous density distribution. For g derived from K and the value is even larger (0.83). The value for f is lower than would be anticipated from the results in Table IV. The indication of these results is that the value for $C_{2,2}$ is probably too large, because of correlations with other coefficients. Nevertheless, the tentative conclusion from these results is that the moment of inertia of the moon is greater than that corresponding to homogeneous density distribution in the lunar interior. Additional analyses are required for precise quantitative determination of the moments of inertia and related parameters for further application.

APPLICATION TO THE TOPOGRAPHY OF THE MOON

The coefficients of the spherical harmonics in the expansion for the gravitational potential can be simply related to the coefficients of surface harmonics, representing variations in the topography of the moon, with assumptions on the density distribution in the moon, as shown by Jeffries (1959). With the assumption of uniform density

distribution in the moon, the equation of the surface can be expressed as

$$a'(\phi, \lambda) = a \left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^n P_{n,m}(\sin \phi) \frac{2n+1}{3} (C_{n,m} \cos m \lambda + S_{n,m} \sin m \lambda) \right]$$

Contours of the topography of the moon, derived from the gravitational field results of Table III, representing differences in radius from the mean value of 1738 km, are shown in figures 1 and 2. The assumption of uniform density distribution implies that the positive contours represent regions of greater mass and the negative contours represent regions of lesser mass than the average. The plots show that the maximum deviations from a spherical shape are about 2.0 km on the portion of the lunar surface facing the earth and about 2.5 km on the far side. The variations in topography are very much dependent on the values of the higher degree and order coefficients, through the polynomials $P_{n,m}(\sin \phi)$, and thus are subject to the uncertainties in these coefficients. Although there is a very slight indication that the maria may represent areas with mass exceeding the mean value and the highlands may represent areas with mass less than the mean value, this indication is not wholly consistent and not sufficiently pronounced to draw any firm conclusions at this stage of the analysis.

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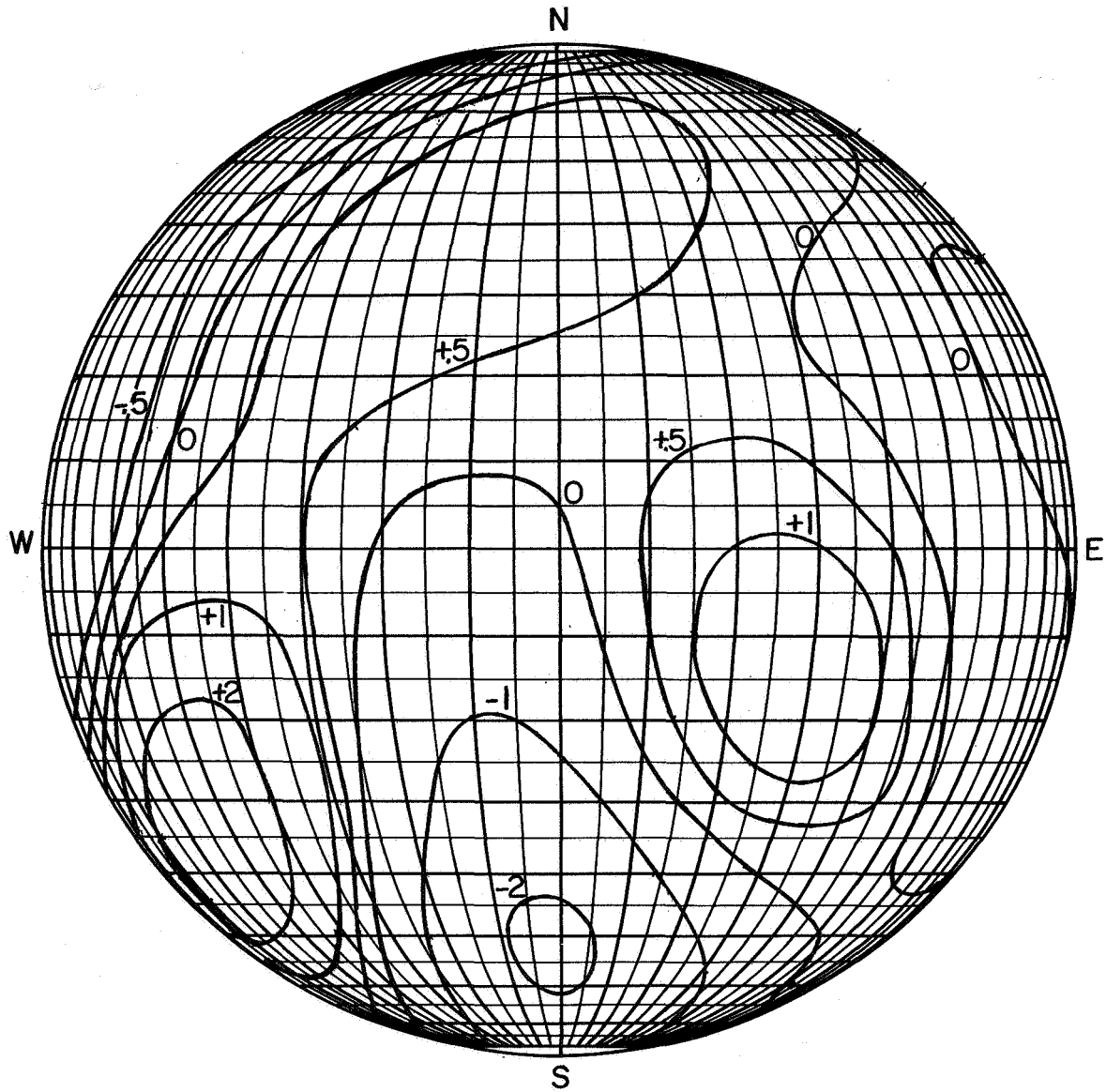


Figure 1.- Contours of Lunar Radii Derived from Gravitational Field Results. (Deviations in km with respect to mean radius of 1738 km.) Hemisphere facing earth.

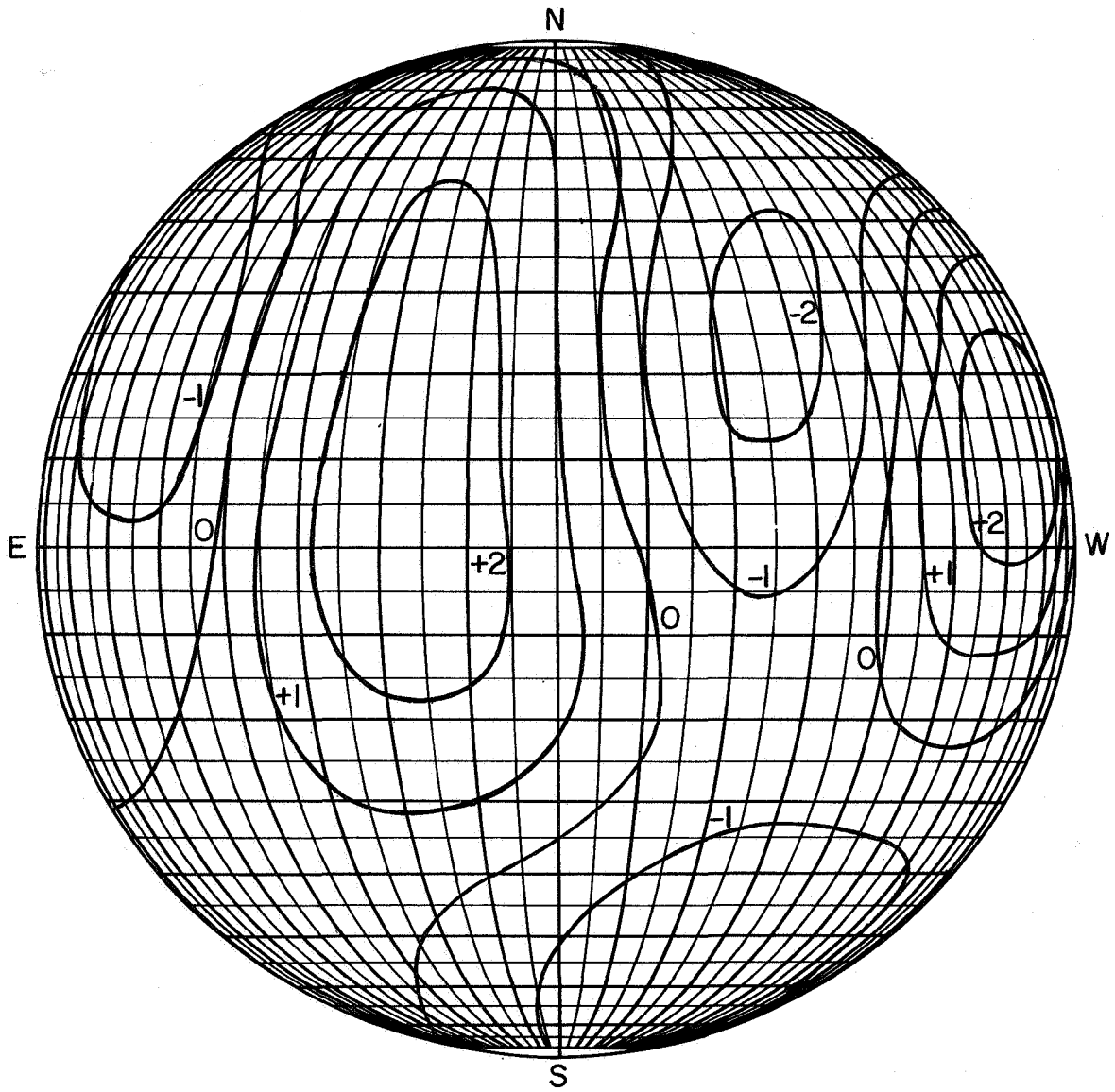


Figure 2.- Contours of Lunar Radii Derived from Gravitational Field Results. (Deviations in km with respect to mean radius of 1738 km.) Hemisphere away from earth.

TABLE I

SUMMARY OF PERTINENT LUNAR ORBITER FLIGHT INFORMATION

Lunar Orbiter Spacecraft	I	II	III
Launch	Aug. 10, 1966 222:18:26	Nov. 6, 1966 310:23:21	Feb. 5, 1967 036:01:17
Injection into Lunar Orbit	Aug. 14, 1966 226:15:34	Nov. 10, 1966 314:20:24	Feb. 8, 1967 039:21:54
Transfer to Photo Orbit	Aug. 21, 1966 233:09:51	Nov. 15, 1966 319:22:58	Feb. 12, 1967 043:18:13
Start Photos	Aug. 22, 1966 234:13:31	Nov. 18, 1966 322:15:24	Feb. 15, 1967 046:10:00
Orbit Adjustment	Aug. 25, 1966 237:16:01	- - -	- - -
End Photos	Aug. 29, 1966 241:13:24	Nov. 25, 1966 329:21:12	Feb. 23, 1967 054:02:11
End Photo Transmission	Sept. 13, 1966	Dec. 6, 1966	March 2, 1967
Inclination Change (to 17.5°)	- - -	Dec. 8, 1966 342:20:36	- - -
Terminated by Commanded Impact on Lunar Surface	Oct. 29, 1966 302:12:30	- - -	- - -

All times are Greenwich Mean Time, in calendar date and in
Day of Year:Hour of Day:Minute.

TABLE II
SUMMARY OF DATA ARCS USED FOR THIS ANALYSIS

Spacecraft, Lunar Orbiter	I		II	III	
Orbit Configuration Identification	1	2	1	1	2
Epoch of Beginning of Data Arc	1966 226:16	1966 260:12	1966 342:20	1967 039:22	1967 043:18
End of Data Arc	230:19	284:16	363:19	043:16	047:10
Total Hours in Data Arc	99	580	503	90	88
Observations used in Data Arc	1330	2080	1768	1320	1176
Approximate Orbital Parameters					
Semi-major axis, a, km	2765	2670	2702	2744	2689
Eccentricity, e	0.30	0.33	0.34	0.29	0.33
Inclination, to lunar equator, deg	≈12	≈12	≈17.5	≈21	≈21

Times are Greenwich Mean Time, in Day of Year: Hour of Day.

TABLE III
PRELIMINARY LUNAR GRAVITATIONAL FIELD COEFFICIENTS

C			S		
n,m	Coeff. x 10 ⁴	Std. Dev. x 10 ⁴	n,m	Coeff. x 10 ⁴	Std. Dev. x 10 ⁴
2,0	-2.2187	.185	--	--	--
2,1	.1250	.049	2,1	.2145	.053
2,2	.3204	.075	2,2	-.3052	.079
3,0	.2943	.029	--	--	--
3,1	.2350	.049	3,1	.3934	.050
3,2	.1933	.027	3,2	.1440	.025
3,3	-.0317	.028	3,3	.0831	.027
4,0	-.0418	.156	--	--	--
4,1	.0163	.055	4,1	.1908	.037
4,2	-.0276	.014	4,2	-.1035	.016
4,3	.0179	.008	4,3	.0076	.009
4,4	-.0014	.005	4,4	-.0147	.005
5,0	--	--	--	--	--
5,1	-.1274	.037	5,1	.1951	.038
5,2	.0809	.007	5,2	-.0015	.006
5,3	-.0102	.003	5,3	.0058	.003
5,4	.0030	.0005	5,4	-.0044	.0005
5,5	-.0009	.0005	5,5	.0017	.0005

TABLE IV

COMPARISON OF PREVIOUS RESULTS PERTAINING
TO MASS DISTRIBUTION IN THE MOON

	Jeffries (1961)	Cook (1959) and Jeffries (1961)	Koziel (1967)	Eckert (1965)
β	6.279×10^{-4}		6.29×10^{-4}	
γ	2.049×10^{-4}		2.31×10^{-4}	
L		5.46×10^{-4}		6.07×10^{-4}
K		1.07×10^{-4}		2.19×10^{-4}
f	0.674	0.804	0.633	0.638
$\frac{L}{\beta}$	0.870		0.965	
$\frac{3}{2} \frac{C}{Ma^2}$	0.522		0.949	
$\frac{K}{\gamma}$				

$$\beta = \frac{C - A}{C}$$

$$\gamma = \frac{B - A}{C}$$

$$L = \frac{3}{2} \frac{C - A}{Ma^2}$$

$$K = \frac{3}{2} \frac{B - A}{Ma^2}$$

$$f = \frac{\beta - \gamma}{\beta} = 1 - \frac{K}{L}$$

$$g = \frac{3}{2} \frac{C}{Ma^2} = \frac{L}{\beta} = \frac{K}{\gamma}$$